**Assignment – 2 - Solution**

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eMasters in Communication Systems, IITK

EE901: Probability and Random Processes

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**Q1. In an experiment, two dice are rolled. Consider the sigma-algebra containing all possible subsets of the sample space. Let X be the larger of the two numbers shown. Compute the list of intervals in the events in sigma algebra map to.**

**Solution:**

The total set of possible outcomes =

X is the larger of the two numbers displayed on the dice.

Hence, the sample space/ =

|  |  |
| --- | --- |
| **-algebra** | **Value of X** |
| (1,1) | 1 |
| (1,2),(2,2),(2,1) | 2 |
| (1,3), (2,3),(3,3),(3,2),(3,1) | 3 |
| (1,4),(2,4),(3,4),(4,4),(4,3),(4,2),(4,1) | 4 |
| (1,5),(2,5),(3,5),(4,5),(5,5),(5,4),(5,3),(5,2),(5,1) | 5 |
| (1,6),(2,6),(3,6),(4,6),(5,6),(6,6),(6,5),(6,4),(6,3),(6,2),(6,1) | 6 |

**Q2:** **For the previous question, compute the corresponding events to the**

**X [2; 4] and X [1].**

**Solution:**

The events corresponding to X ∈ [2; 4] :

{(1,2),(2,2),(2,1),(1,3),(2,3),(3,3),(3,2),(3,1), (1,4),(2,4),(3,4),(4,4),(4,3),(4,2),(4,1)}

The events corresponding to X ∈ [1] :

{(1,1)}

**Q3:** Let be the sample space. Find and plot the CDF of X where:

(a) X() = c (c is a constant).

(b) X() = 1 for A and X() = 2 otherwise, with P(A) =

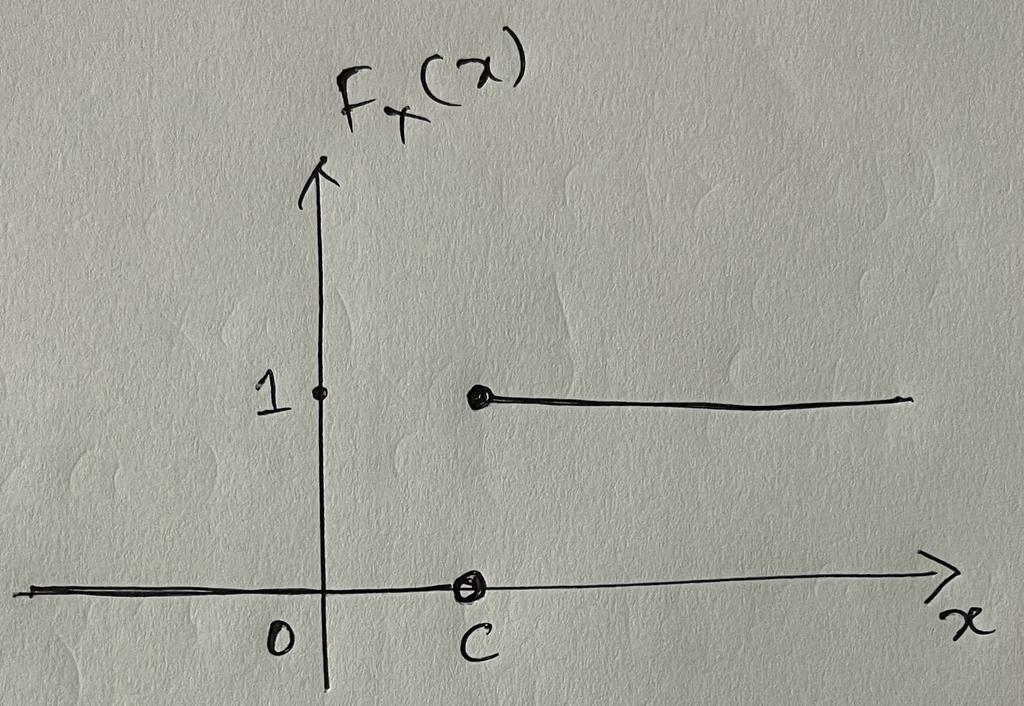
**Solution:**

(a)

|  |  |  |
| --- | --- | --- |
| **x** |  |  |
|  |  | 0 |
|  |  | 1 |

The CDF of X is:

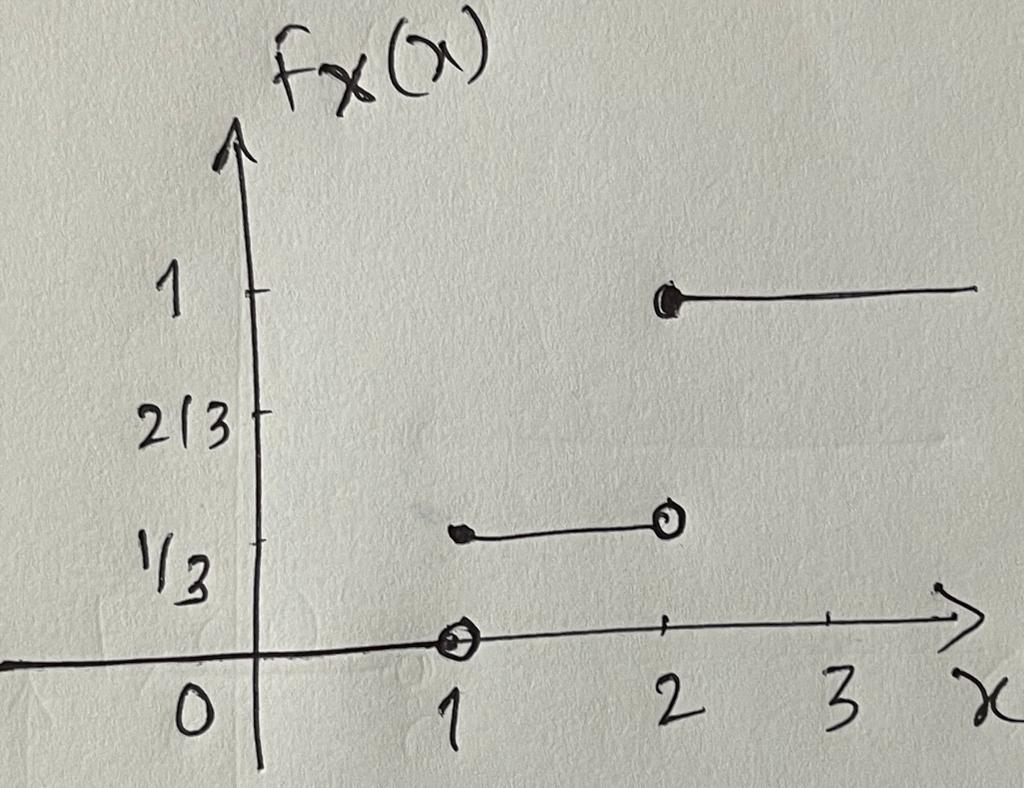
**CDF Plot of X() = c**



(b)

|  |  |  |
| --- | --- | --- |
| **x** |  |  |
|  |  | 0 |
|  |  |  |
|  |  | 1 |

The CDF of X is:

**CDF Plot of X() = 1 for A and X() = 2 otherwise** 

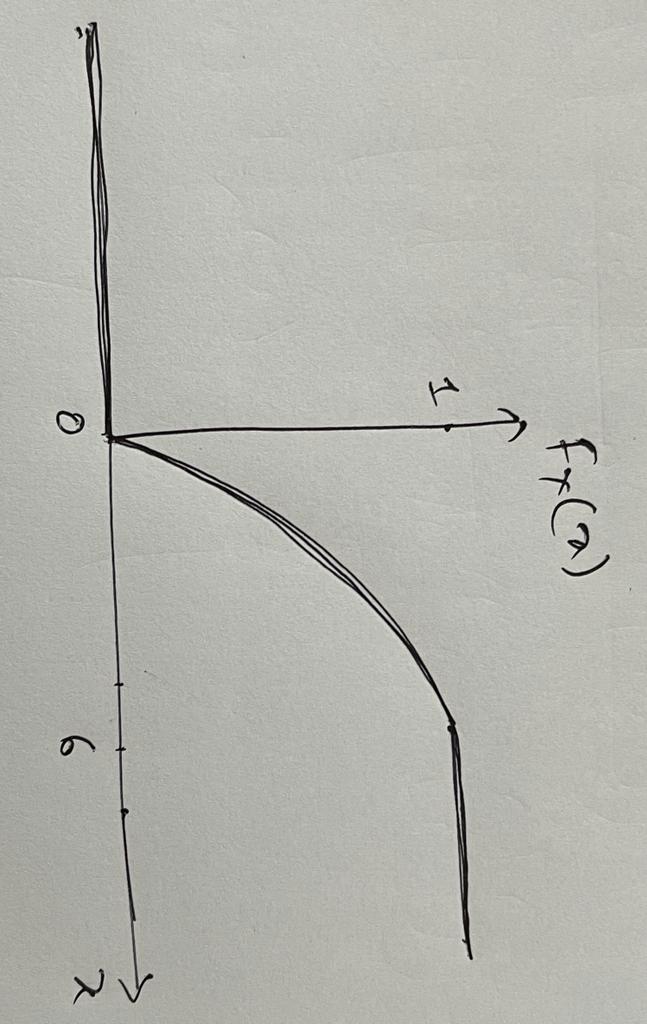
**Q4: Let = [0, 1] be the sample space and let P be a uniform probability measure on it such that P((a, b)) = b-a. Find and plot the CDF of X where X() = 6**

**Solution:**

CDF of X is:

Therefore,

**The CDF Plot of X() = 6**



**Q5:** **Consider the experiment of tossing a coin three times. Let X be the random variable giving the number of heads obtained. We assume that the tosses are independent, and the probability of a head is**

1. What is the range of X?
2. Compute its PMF?
3. Compute the probability that X<2

**Solution:**

**(a) Range of X**

P(H) = 🡺 P(T)

|  |  |  |
| --- | --- | --- |
| Event | Value of X | Probability |
| {TTT} | X=0 |  |
| {HTT,THT, TTH} | X=1 |  |
| {HHT, HTH, THH} | X=2 |  |
| {HHH} | X=3 |  |

So, the outcomes of X are = {0,1,2,3} and hence the range of X is = [0,3]

**(b) PMF**

**(c) Probability of X<2**

**Q6:** **The PDF of a continuous random variable X is given by**

**Find the corresponding CDF Sketch and**

**Solution:**

Let’s look at the area covered by the PDF across the /sample space that should be 1

Hence, the given function is not complaint to PDF principle.

However, if we divide each value of by 2, the area can be complaint to 1.

Therefore,

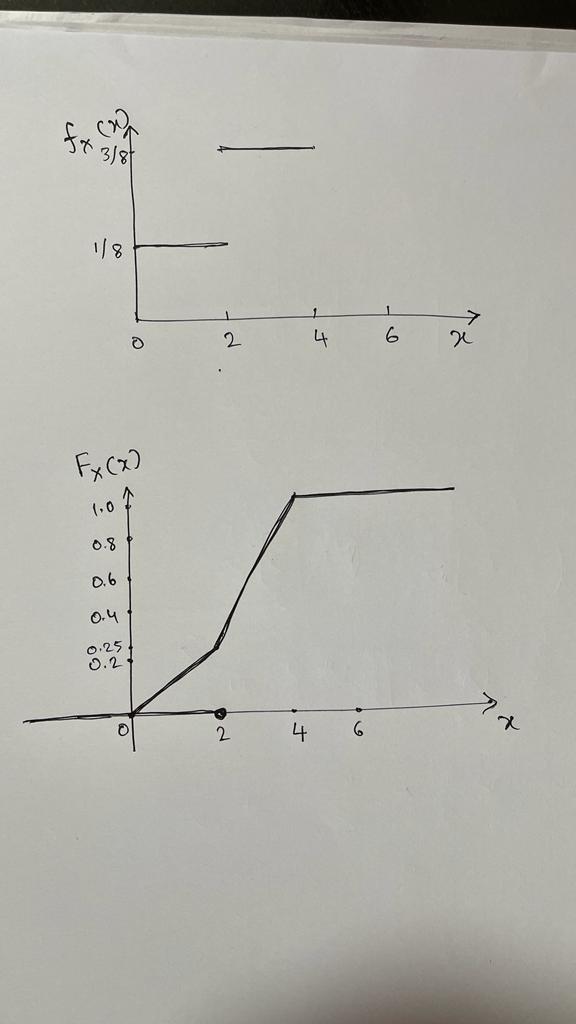
CDF, then is:

**PDF - Plot**

A graph on a white paper

Description automatically generated

**CDF - Plot**



**Q7:** The PDF of a random variable X is given by:

Where is a constant. Find the value of .

**Solution**

**Q8: In an exam, the question paper consists of four multiple-choice questions. A student who does not know any answer, randomly guesses the answers.**

1. What is the probability of getting exactly 3 questions correct?
2. What is the probability that you will write at least 2 questions correctly?

**Solution:**

**(a) Probability of Getting 3 Questions Correct**

If X is the student’s answer, it has two outcomes – correct and wrong for each of the question out of 3 questions. We can apply binomial RV here. Assuming 4 multiple choices for each question, the probability of “correct” outcome is ¼ and “wrong” outcome is ¾.

The binomial distribution

Therefore, the probability of getting exactly 3 questions correct is :

P[X=3]=

**(b) Probability of Getting at least 2 Questions Correct**

= Probability of getting 2, 3, and 4 questions correct

**Q9:** **The amount of time a person waits for a bus is uniformly distributed between zero and 15 minutes. What is the probability that a person waits less than 10 minutes?**

**Solution:**

Let X be the number of minutes a person waits for the bus. The PDF of X, then, is

The probability that a person waits for less than 10 minutes is:

**Q10:** **The time when a bulb of a specific company fails is an exponential random variable with mean .5 years. What is the probability that a bulb will fail in the third year.**

**Solution:**

To solve this problem, we need to use the cumulative distribution function (CDF) of the exponential distribution. The CDF gives us the probability that a random variable is less than or equal to a certain value.

The exponential distribution with mean (μ) has the probability density function (PDF):

f(x) = (1/μ) \* exp(-x/μ) - where x is the time (in years) and exp denotes the exponential function.

The CDF (F(x)) is the integral of the PDF from 0 to x:

F(x) = =

Given that the mean (μ) is 0.5 years, the parameter λ (rate parameter) of the exponential distribution can be calculated as λ = 1/μ = 1/0.5 = 2.

Now, we want to find the probability that a bulb will fail in the third year, which means x = 3. So, we need to evaluate the CDF at x = 3:

F(3) =

To find this integral, we can use integration by parts or look up the CDF of the exponential distribution.

The CDF of the exponential distribution with rate parameter λ is given by:

F(x) = 1 -

Substituting our value of λ = 2, we get:

F(x) = 1 -

Now, for x = 3:

F(3) = 1 -

F(3) = 1 -

Using a calculator, we find that F(3) ≈ 0.9975. So, the probability that a bulb will fail in the third year is approximately 0.9975, or 99.75%.